Contrapositive

way of rephrasing an implication

if p then q

if not q then not p ← contrapositive is equivalent to the original

$\forall x \; P(x) \to Q(x)$ ← original

$\forall x \; \neg Q(x) \to \neg P(x)$ ← contrapositive ✗ NOT a negation

ex) For any integers a,b if $\underbrace{a+b \geq 15}_{p}$ then $\underbrace{a \geq 8 \text{ or } b \geq 8}_{q}$

Prove the claim is true by proving the contrapositive.

The contrapositive is: for any integers a,b if NOT $(a \geq 8 \text{ or } b \geq 8)$

not necessary → then NOT $(a+b \geq 15)$

for any integers a,b if $(a<8 \text{ and } b<8)$ then $a+b<15$.

simplified contrapositive.

Suppose we have integers a,b where $a<8$ and $b<8$.

Since a and b are integers, $a \leq 7$ and $b \leq 7$.

Then, $a+b \leq 14 < 15$. So, $a+b<15$.

☆ only for implication claims ☆

contradiction

We want to prove claim P. (implication, conjunction, etc.)

We create a world in which we assume P to be false.

Then, we show that assumption is impossible to make, because it leads to a contradiction → algebraic facts $(4=5)$
assumptions from problem. $(a = 1.2,$ but $a \in \mathbb{Z})$

so P must be true.

ex) There do not exist any integers a and b such that $18a+6b=1$

P

or $\nexists$ → $\neg\exists\ a,b \in \mathbb{Z}$ s.t. $18a+6b=1$

Suppose for contradiction, the claim is false.

That is, $\cancel{\neg\neg}\exists\ a,b \in \mathbb{Z}$ s.t. $18a+6b=1$

$\exists\ a,b \in \mathbb{Z}$ s.t. $18a+6b=1$

Then, $3a+b = 1/6$

We know $a,b \in \mathbb{Z}$, so $3a+b \in \mathbb{Z}$. But $1/6 \notin \mathbb{Z}$.

This is a contradiction.

Therefore, our original claim was true.